Rigid Plate Transformations to Support PPP and Absolute Positioning in Africa

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SUMMARY

PPP, global RTK and GNSS post-processing services are now used extensively to provide realisations of ITRF and WGS84 globally with a precision of a few centimetres. Unless these instantaneous realisations are transformed to a static or semi-kinematic datum using a suitable kinematic transformation model, repeat surveys using these techniques will result in datum divergence as a function of time arising from the effects of unmodelled tectonic plate motion. Africa has a very sparse CORS infrastructure, and this limitation supports the use of PPP and related techniques. At present, there is no kinematic transformation applied through these services to maintain consistency of coordinate solutions, which account for plate motion.

This paper describes a simple transformation strategy that can be applied on the African continent to enable instantaneous ITRF and WGS84 positions to be transformed to a specified reference epoch with a precision of a few centimeters on a decadal timescale. Adoption of this strategy will enable PPP and absolute ITRF positions to be transformed consistently to a fixed reference epoch for any given location in Africa not subject to localised deformation. The transformation parameters described are tested on a selection of IGS stations in Africa, and a worked example is shown to assist with implementation of the algorithm within software applications.

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1. INTRODUCTION

Precise Point Positioning (PPP) techniques, global RTK services such as OmniSTAR (OmniSTAR, 2010) and StarFire (NavCom, 2010), GNSS post-processing services such as AusPOS (Dawson *et al.*, 2001), NRCan's CSRS-PPP (NRCan, 2004), and OPUS (NOAA, 2010) are now used extensively to provide realisations of ITRF and WGS84 globally with a precision of a few centimetres. Unless these instantaneous realisations are transformed to a static or semi-kinematic datum using a suitable kinematic transformation model, repeat surveys using these techniques will result in datum divergence as a function of time arising from the effects of unmodelled tectonic plate motion. Africa has a very sparse CORS infrastructure, and this limitation supports the use of PPP and related techniques.

This paper describes a simple transformation strategy that can be applied on the African continent to enable instantaneous ITRF and WGS84 positions to be transformed to a specified reference epoch with a precision of a few centimetres. Adoption of this strategy will enable PPP and absolute ITRF positions to be transformed consistently to a fixed reference epoch on a decadal time scale for any given location in Africa not subject to localised deformation. The transformation parameters described are tested on a selection of IGS stations in Africa, and a worked example is shown to assist with implementation of the algorithm within software applications.

2. PLATE MOTION WITHIN AFRICA

The African continent is broadly divided into two major tectonic plates (Figure 1). Most of Africa, west of the East African Rift lies on the Nubian Plate. The Somalian Plate lies East of the African Rift. A very small section of North Africa along the Maghreb coast in Algeria and Morocco lies on the Eurasian Plate and the Dankalia region of Eritrea lies on the Arabian Plate. The Somalian and Nubian Plates are rifting apart at up to 6 mm/yr across the East African Rift (Stamps *et al.*, 2008), and the Nubian Plate is converging with the Eurasian Plate across the Maghreb coastal ranges at ~4 mm/yr (McClusky *et al.*, 2003; Stich *et al.*, 2006).

Analysis of the ITRF2005 solution (Altamimi *et al.*, 2007; IERS, 2010) indicates that away from the African plate boundaries baseline stability between any stable monuments on the Nubian Plate is better than 1.5 mm/yr. However, ITRF site velocities for any location within Africa are between 24 mm and 31 mm/yr due to rigid motion of the African plates over the underlying mantle. These site velocities degrade the accuracy of PPP and absolute positions if the measurement epoch is misinterpreted as the reference epoch for the underlying datum realisation in use at the time (e.g. an epoch 2011 coordinate is misinterpreted as epoch 2002).

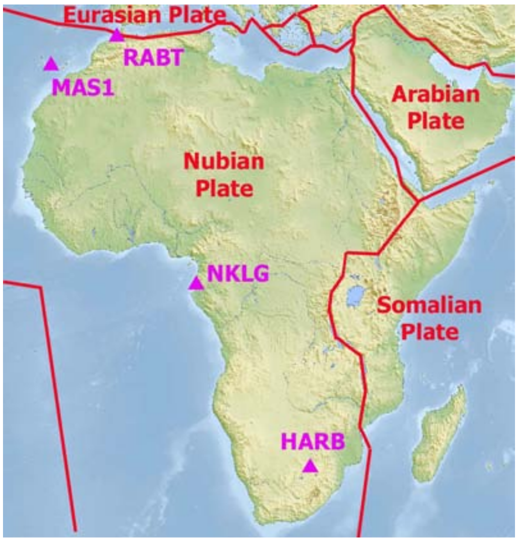

Figure 1. Principal Tectonic Plates in Africa and selected IGS/ITRF stations

3. THREE PARAMETER RIGID PLATE TRANSFORMATION FROM KINEMATIC ITRF TO A FIXED REFERENCE EPOCH

Transformations between a kinematic datum such as ITRF and a static or semi-kinematic datum typically use a 14 parameter model (7 conformal parameters and associated rates of change). For applications where a precision of < 3 cm is acceptable, a simplified three parameter kinematic rigid plate model can be used (Stanaway and Roberts, 2009), if the static datum is aligned with ITRF at a fixed reference epoch.

Rigid Plate movement is conventionally defined by a rotation rate about an Euler Pole (Φ, Λ, and ω), where Φ, Λ are the latitude and longitude of the pole, and ω is the rate of rotation of the plate around the pole in degrees per million years. Equivalent rotation rates about the Cartesian axes (Ω_X, Ω_Y, and Ω_Z) can be computed from the Euler pole definition using equations (1-3) (Φ, Λ, and ω are first converted from decimal degrees to radians)

$$\Omega_X = \mathrm{COS}(\Phi)\,\mathrm{COS}(\Lambda)\omega \qquad (1)$$

$$\Omega_Y = \mathrm{COS}(\Phi)\,\mathrm{SIN}(\Lambda)\omega \qquad (2)$$

$$\Omega_Z = \mathrm{SIN}(\Phi)\omega \qquad (3)$$

For any location on a rigid plate, instantaneous ITRF coordinates can be transformed to a fixed reference epoch using equation (4) (Stanaway and Roberts, 2009).

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + S \cdot \left[\begin{bmatrix} X_t \\ Y_t \\ Z_t \end{bmatrix} + \begin{bmatrix} \Omega_Y Z_t - \Omega_Z Y_t \\ \Omega_Z X_t - \Omega_X Z_t \\ \Omega_X Y_t - \Omega_Y X_t \end{bmatrix} \cdot \left(t_0 - t\right) \cdot 1\text{E-6} \right] \qquad (4)$$

where;

(X_0, Y_0, Z_0) are the ITRF Cartesian coordinates at the reference epoch t_0 (in decimal years),
(X_t, Y_t , Z_t) are instantaneous ITRF Cartesian coordinates at epoch t (epoch in decimal years),
(T_X, T_Y , T_Z) is the translation of the reference frame origin (from ITRF to local system),
(Ω_X, Ω_Y, Ω_Z) are the Cartesian rigid plate/block rotation parameters, and
S is the reference frame scale factor (from ITRF to local).

The translation and scale components are 0 and 1 respectively for any ITRF aligned realisation, however these components do need to be specified for transformation from ITRF into a non-geocentric system.

Assuming that a static realisation of ITRF is used as the basis for a local static datum, then equation (4) can be broken down into the three component equations (5-7) to facilitate software coding.

$$X_0 = X_t + (\Omega_Y Z_t - \Omega_Z Y_t).\left(t_0 - t\right) \cdot 1\text{E-6} \qquad (5)$$

$$Y_0 = Y_t + (\Omega_Z X_t - \Omega_X Z_t).\left(t_0 - t\right) \cdot 1\text{E-6} \qquad (6)$$

$$Z_0 = Z_t + (\Omega_X Y_t - \Omega_Y X_t).\left(t_0 - t\right) \cdot 1\text{E-6} \qquad (7)$$

The ITRF2005 absolute rotation poles for Africa defined by Altamimi *et al.* (2007) are converted to Cartesian format using equations (1-3) (Table 1).

Plate	**Absolute Pole Cartesian angular velocity**		
	Ω_X (Rad/Ma)	**Ω_Y (Rad/Ma)**	**Ω_Z (Rad/Ma)**
Eurasian	-0.000263	-0.002512	0.003791
Nubian	0.000394	-0.002995	0.003594
Somalian	0.000026	-0.003196	0.004344

Table 1. ITRF2005 plate absolute rotation poles in Africa (derived from Altamimi *et al.*, 2007)

4. TESTING THE PARAMETERISATION IN AFRICA

To test the simplified transformation strategy, data from four African CORS stations in the IGS data archive for Day of Year (DOY) 001, 2009 were processed using the NRCan CSRS-PPP service (NRCan, 2004). The PPP results are shown below (Table 2). The estimated positional uncertainty[1] for each 24 hour data file submitted is < 20 mm. Analysis of multiple day solutions can improve the uncertainty to better than 10 mm.

	ITRF2005 at Epoch 2009.0 (metres)		
Site	X	Y	Z
HARB	5084657.641	2670325.206	-2768481.092
MAS1	5439192.218	-1522055.413	2953454.919
NKLG	6287385.757	1071574.631	39132.976
RABT	5255617.667	-631745.605	3546322.612

Table 2. NRCan CSRS-PPP Solutions for Day of Year 1, 2009 (precision < 20mm)

The PPP results were then transformed to 10 different reference epochs between 2000 and 2010 using the three parameter kinematic transformation described in order to verify the stability of the transformation over time. The transformed coordinates were then compared with the predicted ITRF2005 locations at each epoch using the tabulated ITRF2005 site coordinates and site velocities (IERS, 2010).

Site Code	Plate	ITRF2005 at Epoch 2000.0 (metres)			Site Velocity (m/yr)		
		X	Y	Z	V_X	V_Y	V_Z
HARB	Nubian	5084657.639	2670325.018	-2768481.235	-0.0019	0.0190	0.0167
MAS1	Nubian	5439192.240	-1522055.577	2953454.771	-0.0028	0.0179	0.0163
NKLG	Nubian	6287385.799	1071574.422	39132.804	-0.0051	0.0210	0.0193
RABT	Nubian	5255617.716	-631745.773	3546322.471	-0.0080	0.0174	0.0149

Table 3: ITRF2005 Reference Frame sites analysed (IERS, 2010)

Figure 2 below shows the comparison of transformed site coordinates for the selected stations at each epoch with kinematic ITRF coordinates at the same epoch. The plot clearly shows the change in instantaneous ITRF as a function of time, due to the effects of plate tectonics. The change is also a measure of the error in position if the instantaneous coordinates are misinterpreted as those of a fixed reference epoch. For example, an OmniStar-HP position observed in Africa in 2010, interpreted as an Epoch 2000.0 coordinate will have an error of ~ 0.25 metres. Unfortunately, this misinterpretation is widespread. The plot shows that the three parameter technique can maintain coordinate stability of better than 30 mm over a period of a few decades.

[1] Positional Uncertainty is defined as the uncertainty of the coordinates or height of a point, in metres, at the 95% confidence level, with respect to the defined reference frame.

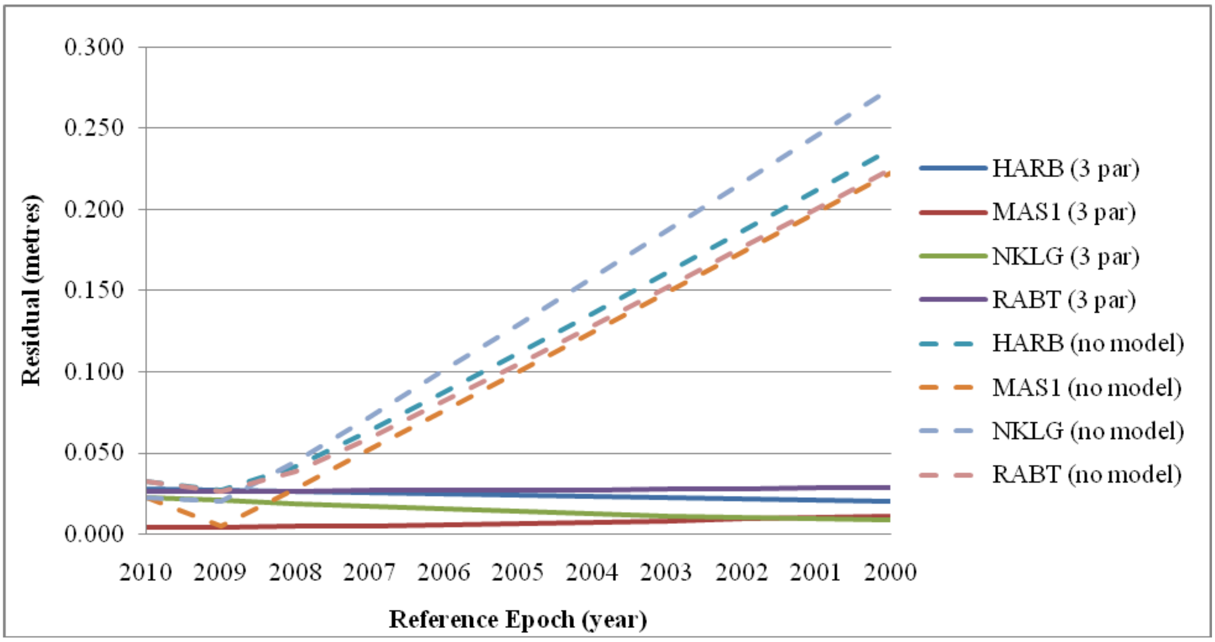

Figure 2. Effect of unmodelled tectonic deformation on ITRF2005 Epoch 2009.0 PPP solutions at different epochs for selected African sites (dashed lines). Solid lines indicate stability of the described 3 parameter rigid-plate transformation for the same sites.

5. LIMITATIONS OF THE THREE PARAMETER MODEL

Theoretically, the three parameter model will not degrade within a perfectly rigid plate. As stated previously however, intraplate deformation in conjunction with monument instability will degrade the precision of the method over time. When this deformation exceeds the tolerances required, a new reference epoch closer to the measurement epoch should be adopted. Alternatively, a deforming plate can be divided into smaller tectonic blocks which are showing evidence of independent motion from the parent tectonic plate. Euler poles for these smaller blocks can then be defined to minimise errors in the modeling.

Within the plate boundary regions (e.g. in the vicinity of the African Rift and in the Maghreb coastal areas) there will be inter-seismic deformation of up to a 3-4 mm year which will not be modelled using a rigid plate transformation model. Again, either more frequent changes in reference epoch, or additional deformation parameters are required to augment the parameterisation.

Larger earthquakes result in coseismic and postseismic deformation of up to a metre or more, and following such events where deformation is evident, a correction (patch) needs to be applied in order to maintain consistency of the datum (Blick *et al.*, 2006). Vertical deformation rates are also not modelled, and in instances where groundwater removal and associated subsidence is occurring, this will affect coordinate repeatability over time.

6. WORKED EXAMPLE

A worked example is described to assist software developers. Data from IGS station MALI (Malindi, Kenya) on the Somalian Plate for DOY 136, 2008 are processed using the NRCan PPP service, however the coordinates need to be transformed to ITRF2005 Epoch 2000 (t_0)

$$t_0 = 2000$$

The epoch of measurement (t) in decimal years is calculated as follows:

$$t = 2008 + 136/366 = 2008.372$$

The PPP service computes coordinates for MALI at epoch 2008.372 as follows:

The estimated coordinates / standard deviations for the mali1360 RINEX file are as follows:

Latitude (ITRF05):	-2 59 45.2780 (dms)	/ 0.003 (m)
Longitude (ITRF05):	40 11 39.8260 (dms)	/ 0.011 (m)
Ellipsoidal Height (ITRF05):	-23.352 (m)	/ 0.020 (m)

The ellipsoidal coordinates are converted to Cartesian format (e.g. using Ordnance Survey, 2010):

$$X_t = 4865366.292$$
$$Y_t = 4110737.666$$
$$Z_t = -331121.514$$

MALI lies on the Somalian Plate, so the parameters (Ω_X, Ω_Y, Ω_Z) are obtained from Table 2:

$$\Omega_X = 0.000026$$
$$\Omega_Y = -0.003196$$
$$\Omega_Z = 0.004344$$

The coordinates of MALI at reference epoch ($t_0 = 2000$) are computed using equations (5-7):

$$X_0 = 4865366.292 + ((-0.003196*-331121.514) – (0.004344*4110737.666)).(2000-2008.372).(1E-6)$$
$$Y_0 = 4110737.666 + ((0.004344*4865366.292) – (0.000026*-331121.514)).(2000-2008.372).(1E-6)$$
$$Z_0 = -331121.514 + ((0.000026*4110737.666) – (-0.003196*4865366.292)).(2000-2008.372).(1E-6)$$

so,

$$X_0 = 4865366.433$$
$$Y_0 = 4110737.489$$
$$Z_0 = -331121.645$$

For comparison, the ITRF2005 at Epoch 2000.0 coordinates for MALI are listed as follows (IERS,2010):

X = 4865366.435 (difference 0.002)
Y = 4110737.495 (difference 0.006)
Z = -331121.653 (difference 0.008)

The difference is within the error estimate for a 24 hr data file processed by the NRCan CSRS-PPP, so is acceptable.

7. CONCLUSION

The simplified parameterisation described can significantly improve the usefulness of PPP solutions, ITRF based post-processing services and global RTK systems such as OmniSTAR within a localised environment (e.g. mining and precision agriculture) where repeatable precision of better than 3 cm is required over a period of a decade or more. Failure to adopt a kinematic transformation strategy to account for global plate motions will result in significant degradation of the positioning quality if these global systems are used repeatedly in a localised area over a period of more than a few years. The simplified transformation method can be easily coded into GNSS algorithms to enable a reference epoch to be chosen by the user. The correct parameter set can be identified using a pseudorange position mapped onto a polygon file of plate boundaries. The very sparse CORS infrastructure in large parts of Africa mitigates against the use of NRTK and static post-processing using local active networks. The three parameter method described will enable global RTK and PPP systems to provide repeatable sub 3 cm positioning in regions of Africa with no CORS coverage over a period of a decade or more.

REFERENCES

Altamimi, Z., Collileux, J., Legrand, J., Garayt, B., and Boucher, C. (2007); ITRF2005: A new release of the International Terrestrial Reference Frame based on time series of station positions and Earth Orientation Parameters, *Journal of Geophysical Research*, Vol. 112, B09401, doi:10.1029/2007JB004949.

Blick, G., Crook, C., Grant, D., and Beavan, J. (2006); Implementation of a Semi-Dynamic Datum for New Zealand, International Association of Geodesy Symposia, Vol. 128, A Window on the Future of Geodesy, *Proceedings of the International Association of Geodesy, IAG General Assembly Sapporo, Japan June 30 – July 11, 2003*.

Dawson, J., Govind, R., and Manning, J. (2001); The AUSLIG Online GPS Processing System (AUSPOS) , Geoscience Australia (Accessed 11th November 2010).

IERS, (2010); http://itrf.ensg.ign.fr/ITRF_solutions/2005/doc/ITRF2005_GPS.SSC.txt (accessed, 11th November, 2010)

McClusky, S., Reilinger, R., Mahmoud, S., Ben Sari, D., and Tealeb, A., (2003); GPS constraints on Africa (Nubia) and Arabia plate motions, *Geophysical Journal International*, Volume 155, Issue 1, pages 126–138.

NavCom, (2010); http://www.navcomtech.com/StarFire/StarFireHome.cfm (accessed, 11th November, 2010)

NOAA, (2010); National Oceanic and Atmospheric Administration OPUS: Online Positioning User Service, (accessed, 11th November, 2010)

NRCan, (2004); Natural Resources Canada, On-line Precise Point Positioning, (accessed, 11th November 2010)

OmniSTAR (2010); http://www.omnistar.com/about.html (accessed, 11th November 2010).

Ordnance Survey, (2010); Converting between 3-D Cartesian and ellipsoidal latitude, longitude and height coordinates (accessed, 11th November, 2010)

Stamps, S., Calais, E., Saria, E., Hartnady, C., Nocquet, J-M., Cynthia J., Ebinger, C., and Fernandes, R., (2008); A kinematic model for the East African Rift, *Geophysical Research Letters*, Vol. 35, L05304, doi:10.1029/2007GL032781.

Stich, D., Serpelloni, E., Mancilla, F., and Morales, J., (2006); Kinematics of the Iberia–Maghreb plate contact from seismic moment tensors and GPS observations, *Tectonophysics*, 426, p.295-317.

Stanaway, R., and Roberts, C. (2009); A Simplified Parameter Transformation Model from ITRF2005 to any Static Geocentric Datum (e.g. GDA94), *Proceedings from International Global Navigation Satellite Systems Society IGNSS Symposium 2009, Surfers Paradise, Qld, Australia 1 – 3 December, 2009.*

BIOGRAPHICAL NOTES

Richard Stanaway is an external part-time PhD student at the School of Surveying and Spatial Information Systems at the University of New South Wales, Sydney, Australia. He is also the director of Quickclose, a company which specialises in surveying software, geodetic datum analysis and high precision GNSS support for industry. After graduating from the Queensland University of Technology, Brisbane, Australia with a surveying degree in 1998, he worked in Papua New Guinea (PNG) as a geodetic and mining surveyor. In 2000, he expanded the Australian National University's (ANU) geodynamic monitoring network in Papua New Guinea. This work lead him to completing a MPhil at ANU in 2004 that investigated the feasibility of a kinematic datum in PNG. During his time at ANU, he also contributed to geodynamics studies in Antarctica and South Australia. In recent years he has provided geodetic support for mining and petroleum operations in PNG and geodetic validation for airborne laser scanning surveys in both Australia and PNG.

Craig Roberts is a Senior Lecturer in Surveying/ GPS/ Geodesy at the University of New South Wales, Sydney, Australia. He has lectured at RMIT University in Melbourne for two years. He graduated from the University of South Australia with a Bachelor of Surveying in 1988. He began his career as a private surveyor in Adelaide and has since worked as a Geodetic Engineer at UNAVCO, USA involved with GPS for geodynamic studies in Nepal, Ethiopia, Argentina and Indonesia. He worked as a scientific assistant at the GeoForschungsZentrum, Germany where his main focus was orbit determination and prediction for a number of geodetic research satellites. He completed his PhD thesis in March 2002 supervised by Prof. Chris Rizos. His current research interests involve leveraging CORS infrastructure for practical application to surveying and spatial information.

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