

# Rigid Plate Transformations to Support PPP and Absolute Positioning in Africa

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FIG Working Week, Marrakech, Morocco, 18-22 May 2011





# CORS Distribution in Africa

Sparse GNSS CORS  
infrastructure  
overcome by use of  
PPP and Global  
Differential Services

image: Centro GNSS de Canarias  
[www.canarygnsscenter.org](http://www.canarygnsscenter.org)

**Natural Resources Canada**  
www.nrcan.gc.ca

**Canadian Spatial Reference System**

**Canadian Spatial Reference System**

CSRS-PPP

CSRS-PPP is an on-line application for GPS data post-processing that a submit observation data over the Internet and recover, using precise G information, enhanced positioning precisions in the Canadian Spatial Ref (CSRS) and the International Terrestrial Reference Frame (ITRF).

Select RINEX Observation File

(Name: use only Western Roman alphanumerics, including hyphen and underscore; (Compression: none or zip (.zip), gzip (.gz) or UNIX Compress (.Z))  
(Format: RINEX or Compact RINEX (Hatanaka))

Select Mode of Processing  
Static  Kinematic

Select Reference System  
NAD83(CSRS)  ITRF (Epoch of GPS data)

Enter/Change E-Mail to which results will be sent  
richard.stanaway@quickclose.com.au

Date Modified: 2009-02-25

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**OmniSTAR (RTK)**

Station	Latitude (DMS)	Longitude (DMS)	Ellipsoidal Height(m)	Derived Above Geoid Height(m)
RABT	33 59 53.17590	-6 51 15.43821	90.153	44.805

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~~~~~ rbt0010 ~~~~~

The estimated coordinates / standard deviations for the rbt0010 RINEX file are as follow:

Latitude (ITRF05): 33 59 53.1751 (dms) / 0.003 (m)  
 Longitude (ITRF05): -6 51 15.4384 (dms) / 0.009 (m)  
 Ellipsoidal Height (ITRF05): 90.110 (m) / 0.019 (m)

UTM (North) Northing: 3764021.294m Easting: 698173.709m Zone: 29 Scale Factor: 1.00008

AUSPOS Version 2.00

Topic Home Home > Earth Monitoring and Reference Systems > Geodesy and Global Navigation Systems >

Geodesy and Global Navigation Systems

Basics Geodetic Techniques Global Navigation Satellite System Networks Geodetic Datums Astronomical Information

Number of RINEX files  Submit RINEX using  upload  ftp  
 File Name Height (m) Antenna Type  
 Your Email Address:

## 3.2 Geodetic, GRS80 Ellipsoid, ITRF2005

Geoid-ellipsoidal separations, in this section, are computed using a spherical harmonic synthesis of the global EGM2008 geoid. More information on the EGM2008 geoid can be found at <http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/>

| Station | Latitude (DMS) | Longitude (DMS) | Ellipsoidal Height(m) | Derived Above Geoid Height(m) |
|---------|----------------|-----------------|-----------------------|-------------------------------|
| RABT    | 33 59 53.17590 | -6 51 15.43821  | 90.153                | 44.805                        |

## 4.1 Coordinate Precision - Geodetic, One Sigma

| Station | $\sigma$ East (m) | $\sigma$ North (m) | $\sigma$ Up (m) |
|---------|-------------------|--------------------|-----------------|
| RABT    | 0.001             | 0.002              | 0.003           |

## ITRF positioning services (examples)

# Kinematic coordinates illustrated

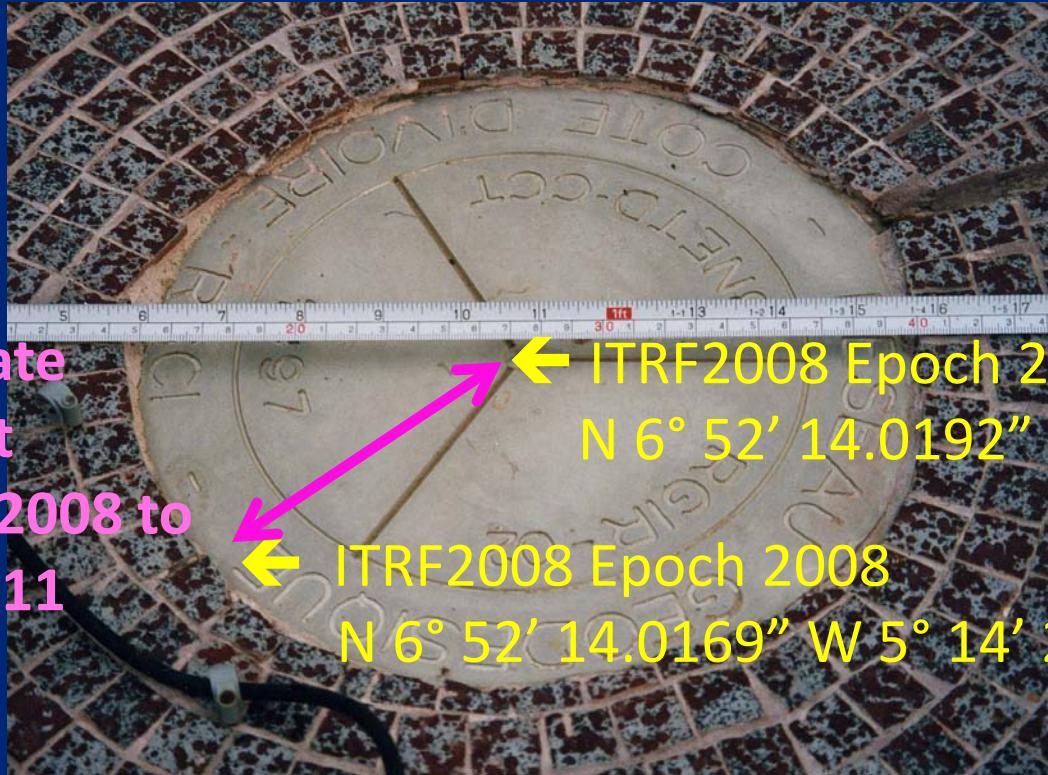


← ITRF2008 Epoch 2008  
N  $6^{\circ} 52' 14.0169''$  W  $5^{\circ} 14' 24.3345''$

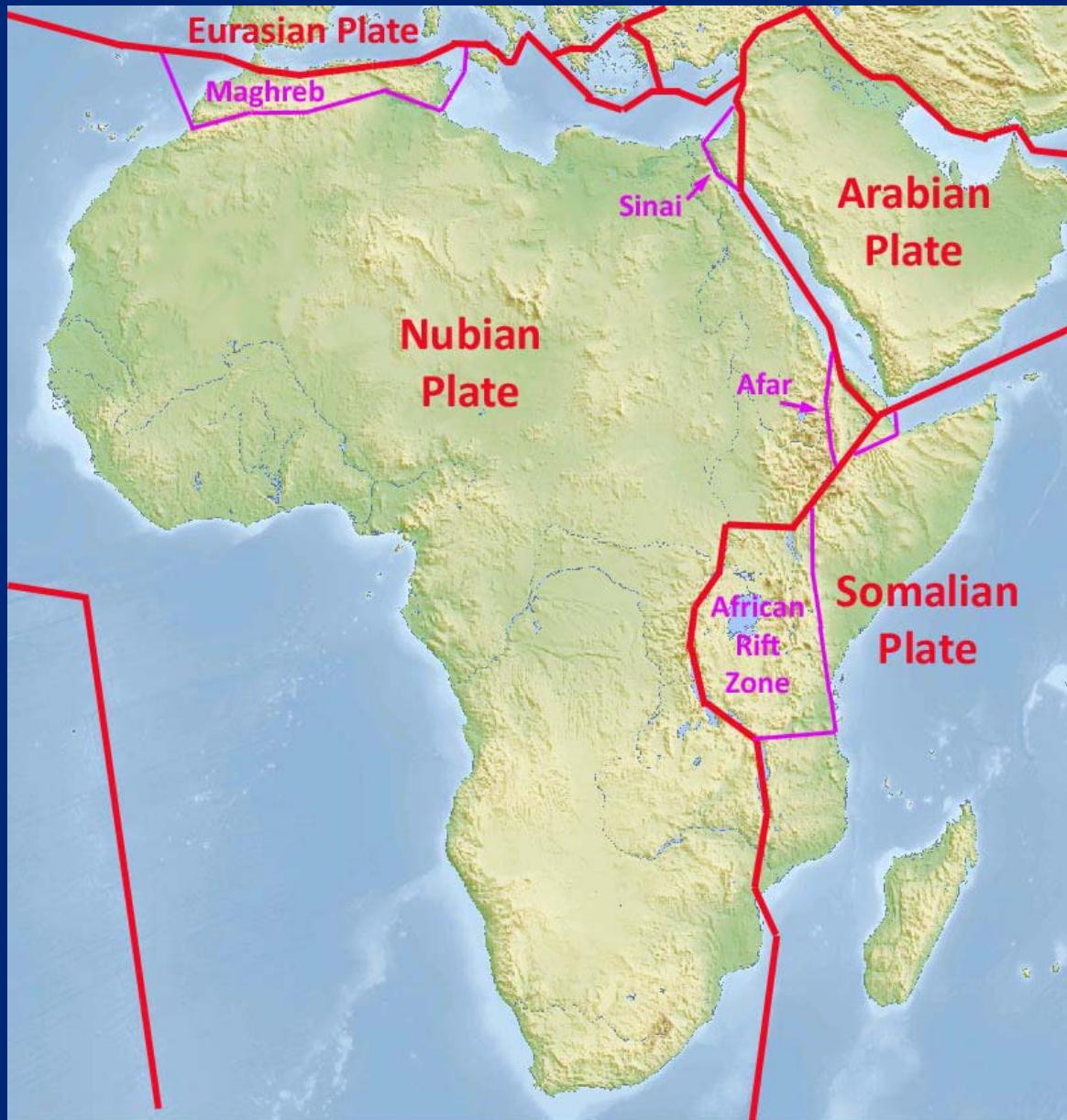
YKRO – IGS Station  
(Yamoussoukro, Cote d' Ivorie)  
From IGS web-site

## Kinematic coordinates illustrated

Nubian Plate  
movement  
1 January 2008 to  
22 May 2011  
109 mm!

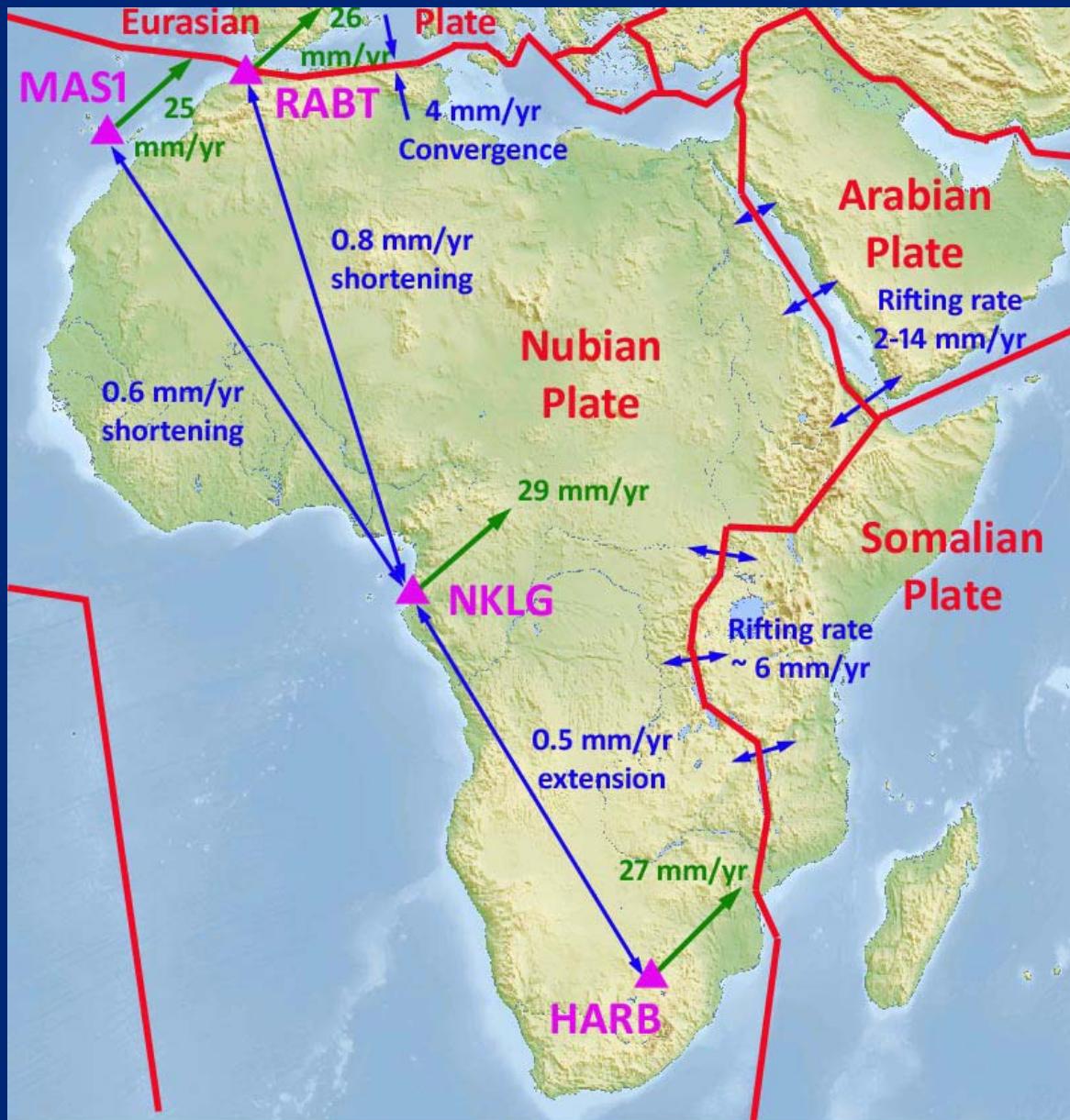


YKRO – IGS Station  
(Yamoussoukro, Cote d' Ivorie)  
From IGS web-site

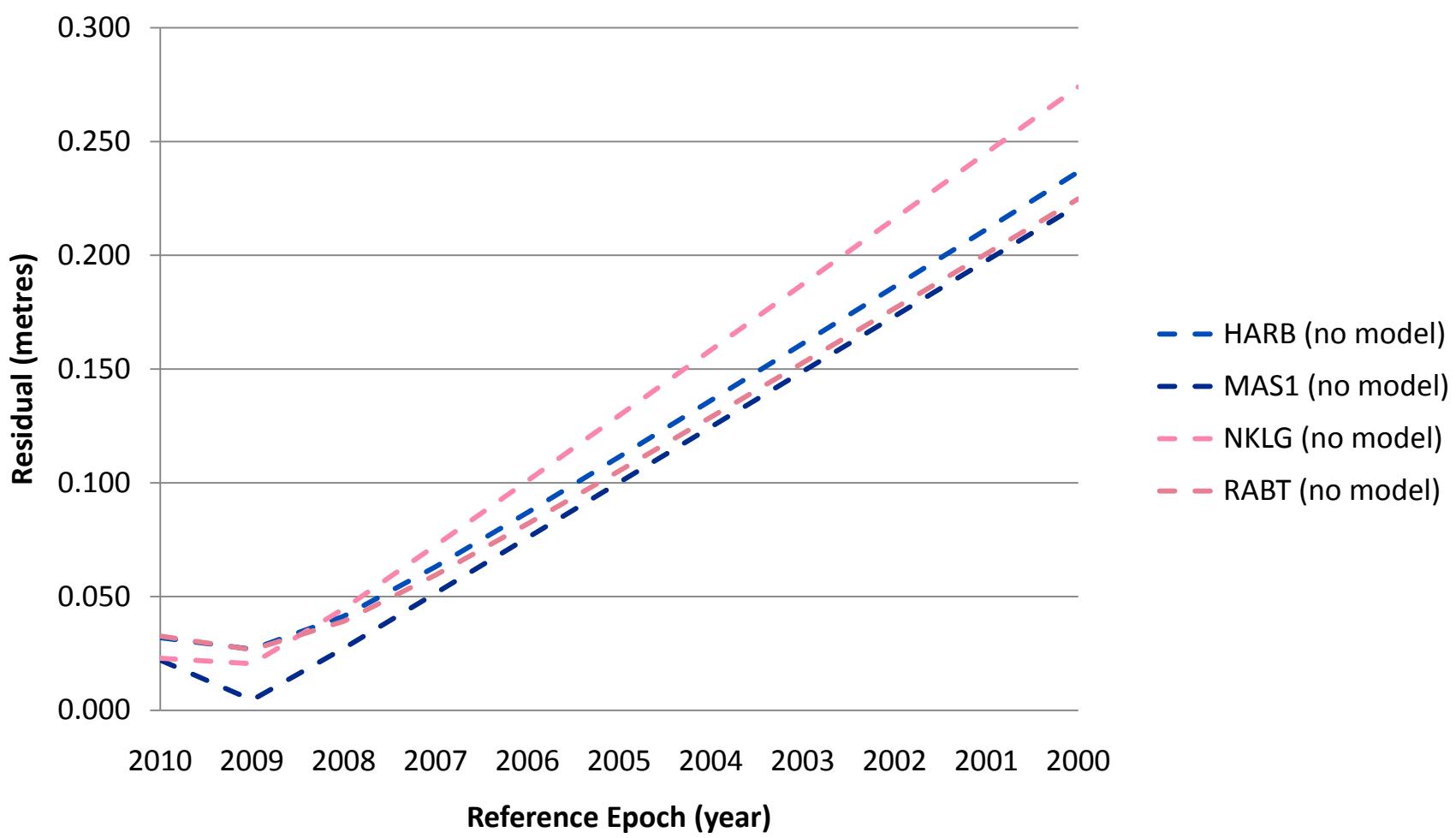


## Principal Plates and Plate Boundaries in Africa

# Stability of the Nubian Plate



Deformation rates computed  
from ITRF2008 GPS SSC Solution  
<http://itrf.ensg.ign.fr>



## Divergence between ITRF (Epoch 2009) and ITRF (epoch of measurement)

# Nubian and Arabian Plate Boundary today

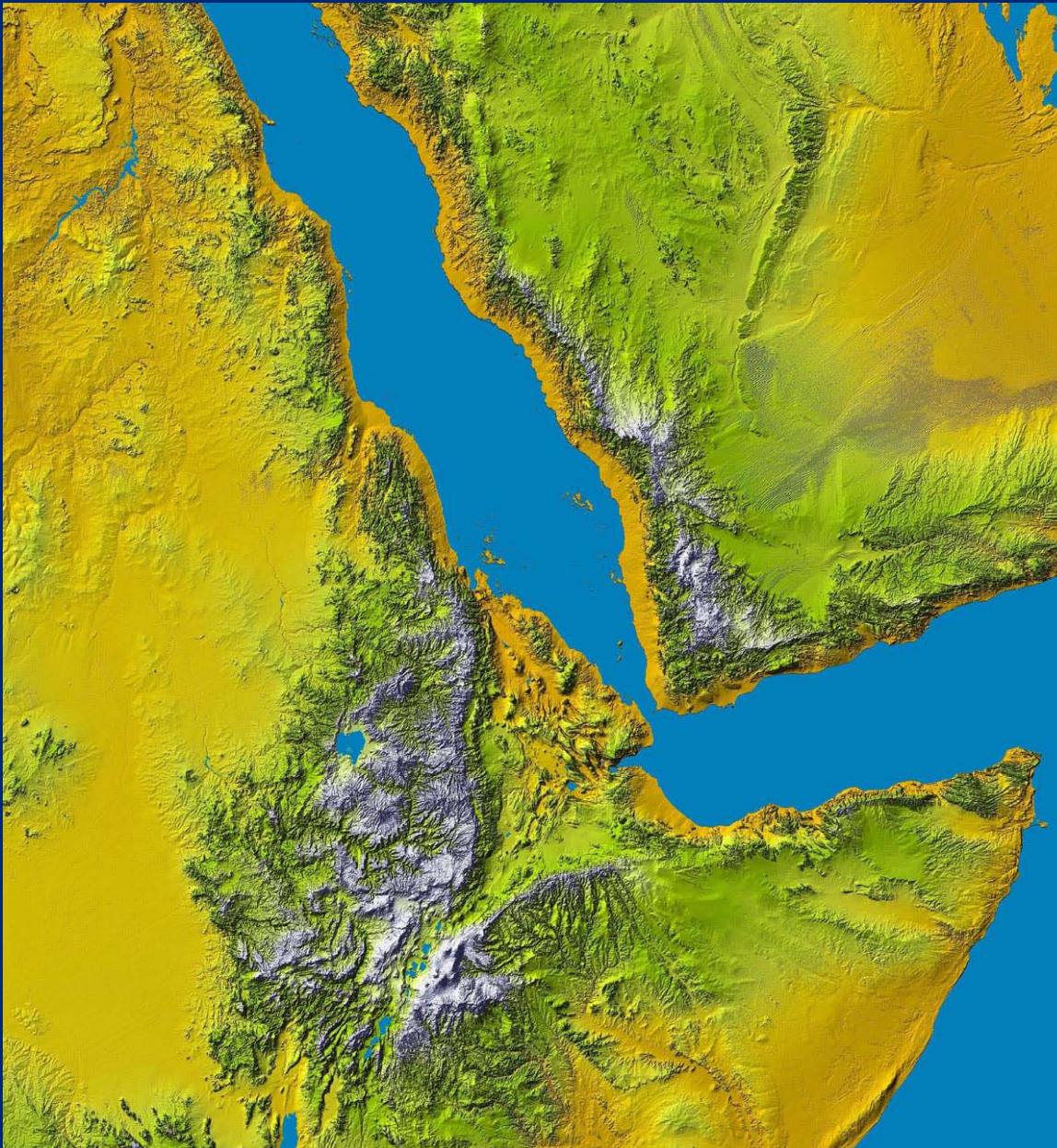
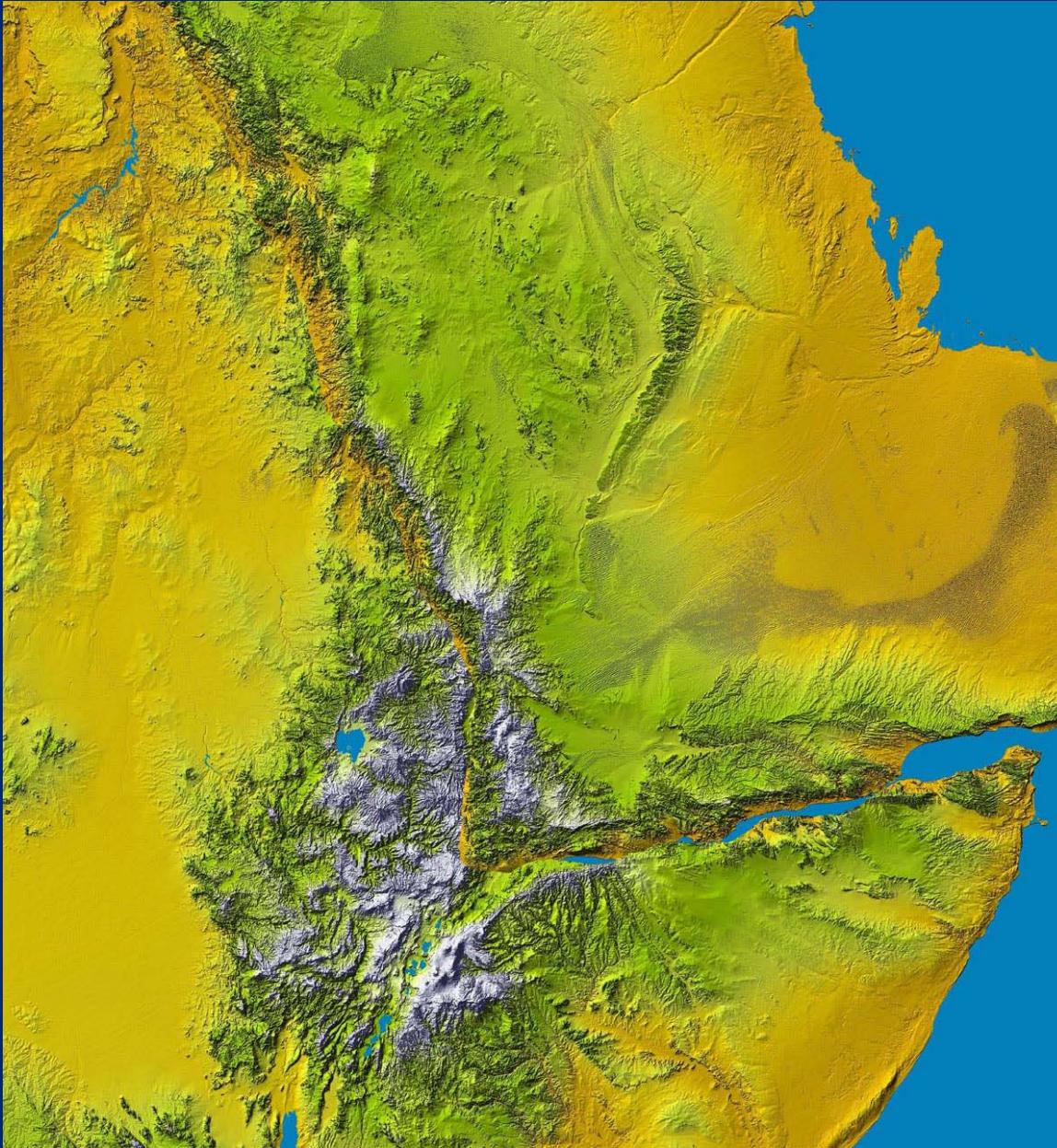


image:  
JPL NASA SRTM

**Nubian and  
Arabian Plate  
Boundary  
30 Ma**



$$\Omega_x = \cos(\Phi) \cos(\Lambda) \omega$$

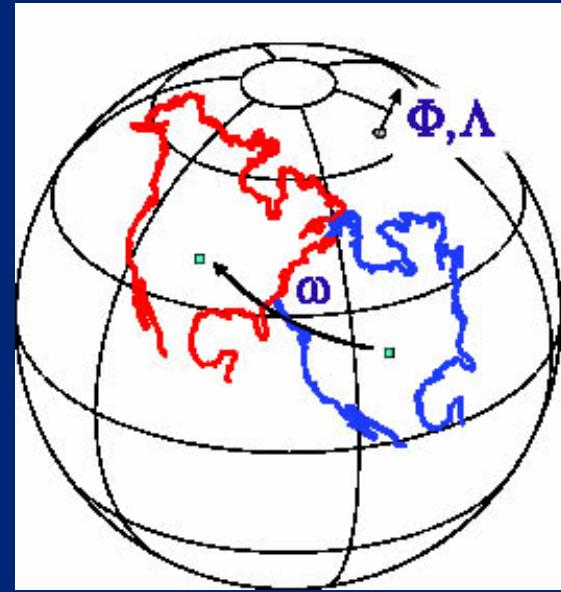
## Rigid Plate Model

Euler Poles to Cartesian rotation rates

$$\Omega_x = \cos(\Phi) \cos(\Lambda) \omega$$

$$\Omega_y = \cos(\Phi) \sin(\Lambda) \omega$$

$$\Omega_z = \sin(\Phi) \omega$$



ITRF2005 African plate parameters  
(Altamimi *et al.* 2007)

| Plate   | Euler pole of rotation |               |                 | Equivalent Cartesian angular velocity |                        |                        |
|---------|------------------------|---------------|-----------------|---------------------------------------|------------------------|------------------------|
|         | $\Phi$ (°)             | $\Lambda$ (°) | $\omega$ (°/Ma) | $\Omega_x$<br>(Rad/Ma)                | $\Omega_y$<br>(Rad/Ma) | $\Omega_z$<br>(Rad/Ma) |
| Arabia  | 49.6                   | 5.1           | 0.579           | 0.006518                              | 0.000577               | 0.007700               |
| Eurasia | 56.3                   | -96.0         | 0.261           | -0.000263                             | -0.002512              | 0.003791               |
| Nubia   | 50.0                   | -82.5         | 0.269           | 0.000394                              | -0.002995              | 0.003594               |
| Somalia | 53.7                   | -89.5         | 0.309           | 0.000026                              | -0.003196              | 0.004344               |

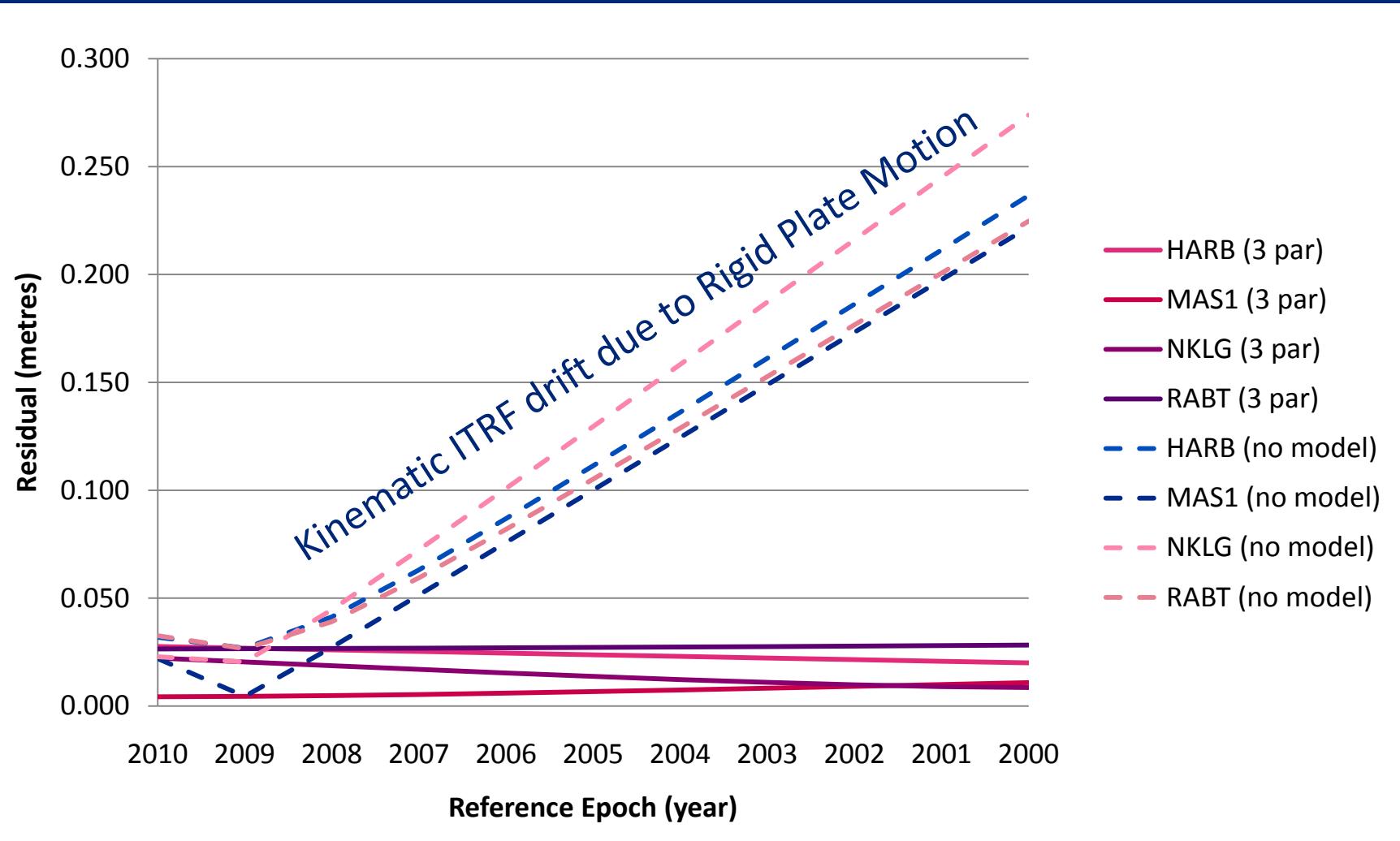
## Kinematic to Static transformation

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + S \cdot \begin{bmatrix} X_t \\ Y_t \\ Z_t \end{bmatrix} + \begin{bmatrix} \Omega_Y Z_t - \Omega_Z Y_t \\ \Omega_Z X_t - \Omega_X Z_t \\ \Omega_X Y_t - \Omega_Y X_t \end{bmatrix} \cdot (t_0 - t) \cdot 1E-6$$

↑ Local frame translation & scale (only if required)      ↑ Plate rotation parameters  
 “Static” coordinates at reference epoch      “Measured” ITRF coordinates  
 ↑ reference epoch      ↑ measurement epoch

$$\begin{aligned}
 X_0 &= X_t + (\Omega_Y Z_t - \Omega_Z Y_t) \cdot (t_0 - t) \cdot 1E-6 \\
 Y_0 &= Y_t + (\Omega_Z X_t - \Omega_X Z_t) \cdot (t_0 - t) \cdot 1E-6 \\
 Z_0 &= Z_t + (\Omega_X Y_t - \Omega_Y X_t) \cdot (t_0 - t) \cdot 1E-6
 \end{aligned}$$

**Simplified  
3-parameter equations  
Kinematic ITRF to  
Static ITRF  
(no scale or translation  
parameters)**



**Improved coordinate consistency using a  
3 parameter rigid plate transformation**

## Limitations of a Rigid Plate Model

Intraplate deformation not accounted for  
(usually small magnitude < 1 mm/yr anyway)

Fails near plate boundaries  
(requires additional modelling of locked faults)

Coseismic and Postseismic deformation not  
modelled

Thank you

Merci