Four Dimensional Deformation Modelling, the link between International, Regional and Local Reference Frames

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SUMMARY

The anticipated improvement in precision and proliferation of GNSS and GIS is dependent upon consistency between ITRF and a local reference frame which defines fixed land boundaries, services, physical infrastructure and machine control (e.g. precision agriculture, automated mining and navigation). This is only achievable through use of a suitable transformation and deformation model to relate the International Terrestrial Reference Frame (ITRF) and closely aligned GNSS reference frames such as WGS84 to a local reference frame at any epoch.

Regional reference frames are typically defined as a regional densification of the ITRF. These reference frames are necessarily kinematic in nature to account for tectonic and site specific deformation. GNSS data processing (e.g. point positioning and baseline processing) are performed within these kinematic reference frames in order to maintain consistency with GNSS orbit products. Time dependent transformations and deformation models are subsequently required to relate these ITRF coordinates to a specified fixed epoch of realisation of a local reference frame.

This paper compares several different approaches to kinematic reference frame transformations and deformation models. The method and strategy used depends upon the precision required and the tectonic setting of the local reference frame. A novel approach to high precision modelling in complex deforming zones with frequent episodes of seismic deformation is presented.

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1. INTRODUCTION

GNSS positioning is intrinsically done within a geocentric reference frame closely aligned with the International Terrestrial Reference Frame (ITRF) e.g. WGS84 and IGS08 for the GPS system. GNSS reference frames are necessarily kinematic in nature to prevent degradation of orbit products arising from relative deformation of GNSS tracking networks as a result of plate tectonics and localised deformation.

The kinematic nature of GNSS positions within ITRF and WGS84 is poorly understood by land surveyors and other spatial science professionals. For example, ad hoc realisations of ITRF derived from Precise Point Positioning services (PPP) (e.g. NrCan) or global RTK (e.g. OmniSTAR, Starfire) and post-processing services (e.g. AusPOS, OPUS) are often used as a basis for an operational datum. This approach is commonplace in countries or regions with a sparse or non-existent network of geodetic monuments and Continuosly Operating Reference Stations (CORS). Such ad hoc realisations of ITRF fixed at a different reference epochs are inconsistent. Repeated use of PPP is further complicated by interseismic tectonic deformation which can be up to 100 $mmyr^{-1}$ in magnitude, and seismic deformation. Unless subsequent positions are corrected for tectonic deformation, continued use of PPP will degrade the precision of localised geodetic networks as a function of time. The implications of this are quite significant considering that global RTK services are often used for precision navigation and steering applications which are typically defined by ground-fixed coordinates (e.g. precision agriculture, automated mining and high precision navigation applications).

The use of CORS and long GNSS baseline processing is also adversely affected by global and local deformations. For example, Network Real Time Kinematic (NRTK) GNSS requires 15 mm a priori precision of the CORS stations (Ramm and Hale, 2004) in order to correctly model tropospheric and orbit biases. This precision is difficult to attain in a deforming zone. Another more sinister effect is that of rigid plate rotation of a geodetic network where no changes in length of GNSS baselines are evident (see Fig. 2). In order for positions from precision GNSS techniques to maintain consistency within a static local reference system, a deformation model or time dependent transformation strategy is required to relate instantaneous (kinematic or dynamic) ITRF coordinates to the local system.

This paper describes a novel approach for the handling of complex global and local deformations when GNSS techniques are used for high precision positioning, with a view to maintaining coordinate repeatability within a local reference frame. The characteristics of different types of reference frames, modes of deformation, positioning tolerances and deformation models are also discussed.

2. HIERARCHY OF TERRESTRIAL REFERENCE FRAMES

2.1 Global Reference Frames

Global reference frames define the fundamental basis for geodetic coordinates and their rates of change for any location with respect to the Earth. Conventionally, the centre of mass of the Earth (the geocentre) is used as the fundamental origin for a global reference system, with the z-axis aligned with the Conventional Terrestrial Pole (CTP), the x-axis aligned with International Reference Meridian (Greenwich) at the equator, and the y-axis along the equator at 90 degrees East (IERS, 2010). Locations on the Earth's surface are moving due to the effect of plate tectonics, so a No-Net-Rotation (NNR) condition is defined in which the angular momenta of all tectonic plates and deforming zones sum to zero (Argus and Gordon, 1991). The NNR condition implicitly defines the motion of features on the Earth's surface with respect to the underlying mantle, which is considered to be coupled with the Earth's rotation on geological time scales. Coordinates in this system are often considered to be "Absolute".

ITRF is considered to be the fundamental realisation of a NNR terrestrial reference system and is defined by the coordinates of a combination of space geodetic sensor monuments and their site velocities around the Earth. ITRF forms the basis for many modern regional and local reference frames. Individual space geodetic techniques define technique specific reference frames (e.g. IGS08 for GPS, WGS84(G1150) for GPS, SLRF2008 for SLR, and GTRF for Galileo), however these are all constrained by ITRF. The current realisation of ITRF is ITRF2008 (Altamimi *et al.*, 2011).

The principal characteristic of a kinematic reference frame such as ITRF is that the coordinates of Earth-fixed features change by up to several centimetres a year due to the effects of plate tectonics. In addition, major earthquakes can result in almost instantaneous coordinate changes of up to several metres. Because GNSS analysis techniques intrinsically use IGS orbit products, the coordinates of GNSS reference stations should be realised by the most recent epoch of ITRF in order to prevent errors in analysis, particularly within tectonically active regions and for long baseline processing. Constantly changing coordinates however, are impractical for most end users. For example, it is very difficult to integrate or combine spatial data collected at different epochs of measurement (e.g. laser scanned point clouds and cadastral data) (Stanaway and Roberts, 2011).

2.2 Regional Reference Frames

Regional reference frames are realised by a denser network of geodetic tracking stations under the aegis of regional collaboration between national geodetic agencies. In regions which are dominated by a single and stable tectonic plate, a plate fixed condition is often used in preference to the NNR condition in order to minimise site velocities of the network stations. The relationship between plate-fixed regional reference frames (e.g. EUREF and NAD83) and ITRF is defined by a 14 parameter conformal transformation (7 parameters at the reference epoch and their rates of change). Regional frames which encompass a variety of tectonic plates adopt a NNR approach and are fully consistent with ITRF (e.g. APREF). Regional reference frames form the basis for national and local reference frames.

2.3 Local Reference Frames

Local reference frames (classically known as geodetic datums) are typically defined by static coordinates of a network of fiducial or zero order geodetic monuments (e.g. OSGB36, GDA94, NAD83, NZGD2000 and IGM95). Local reference frames are the basis for most topographic maps, navigation, GIS, planning, asset management, environmental and cadastral surveys. Over the last twenty years, many developed nations have adopted different realisations of ITRF or regional reference frames as the basis for their national datums, however many jurisdictions still use non-geocentric (e.g. astro-geodetic) and assumed datums. Any changes of coordinates in a static local frame are usually due to improved observations and subsequent network readjustments. The coordinate changes reflect better positional uncertainty and convergence towards their true values rather than any physical movement of the monument. Static reference frames or datums have been sustainable in tectonically stable regions (e.g. Great Britain, Southern Africa, Brazil, Eastern USA, Australia and Southern India).

The defining characteristic of static local reference frames is that the coordinates of the fiducial network which defines the frame, are fixed at a specific reference epoch. Significant improvement in GNSS positioning precision over the last twenty years has highlighted deficiencies of static geodetic datums located within tectonically active deforming zones (e.g. Western USA, Japan, Indonesia, Chile, and New Zealand). Within these deforming zones the strategy of holding coordinates of primary geodetic monuments fixed has resulted in degradation of the geodetic networks due to rapid deformation within the network (up to 50 $mmyr^{-1}$) due to tectonic deformation. The Western USA states (Snay, 1999) and New Zealand (Blick *et al.*, 2005) were among the first jurisdictions to adopt deformation models to compute relative deformation across their geodetic networks to mitigate these tectonic effects. The deformation models enable geodetic observations at the epoch of measurement to be adjusted to a fixed reference epoch (forming quasi-observations) using the deformation model. This approach enables contemporary geodetic measurements to be used within a static coordinate framework. Datums that incorporate deformation models in this way are referred to as semi-dynamic datums. Even within tectonically stable geodetic networks, rigid plate rotation and the drift of the local frame away from ITRF result in inconsistent positioning if time dependent transformations are not used with GNSS positioning (Dawson and Woods, 2010).

3. CHARACTERISATION OF DEFORMATION

The spatial and temporal nature of deformation of reference frames and geodetic monuments should be characterised before the different approaches to modelling can be discussed in any detail. Deformation can be either apparent or real. Apparent deformation is an artefact of geodetic analysis. Real deformation can be periodic (e.g. tidal), site specific (e.g. monument instability), or tectonic in nature. Glacial isostatic adjustment (GIA) is treated in this paper as a tectonic effect in order to simplify the modelling approach, although GIA is not tectonic in origin.

Tectonic deformation is the primary focus of this study and is divided into two components; (1) secular deformation, which broadly encompasses rigid plate motion and interseismic deformation within deforming zones, and, (2) non-secular deformation which includes episodic, non-linear and unpredictable deformation effects arising from seismic activity. The temporal and spatial domian of these effects is illustrated (Fig. 1).

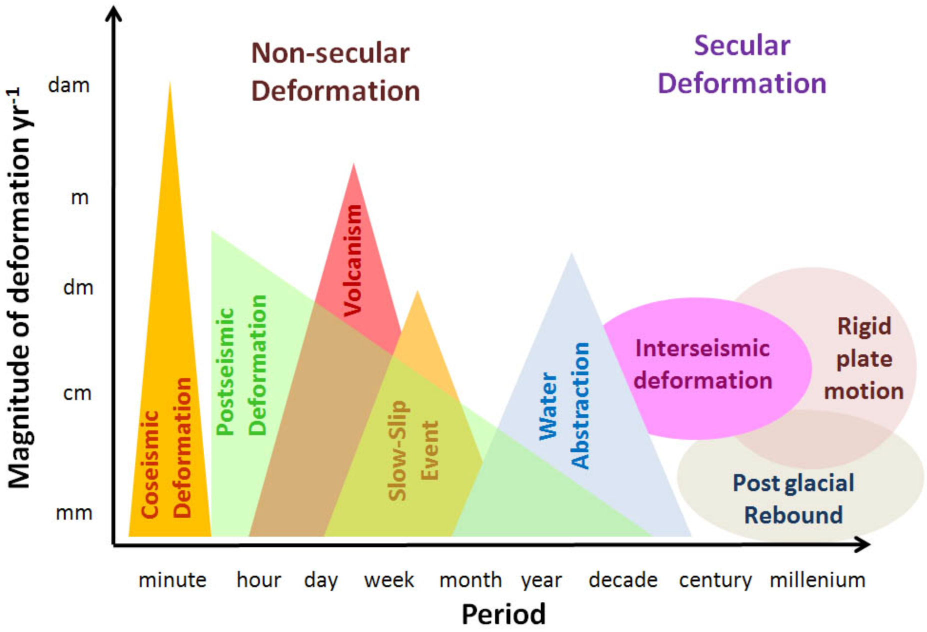

Figure 1. Temporal and spatial domain of terrestrial deformation

3.1 Apparent Deformation

Apparent deformation effects are usually artefacts of geodetic analysis and do not represent real deformation of the monument and are often a major source of error in precise geodetic analysis and site velocity estimation. Spectral analysis of a site's time series, precision tie and co-location surveys can usually identify these effects and improve the precision of the estimation of site position and velocity. Most of the apparent deformation is manifest in the station height. These effects should be mitigated prior to the estimation of site coordinates and velocities to ensure that the site velocity actually represents movement of the underlying bedrock or regolith and by inference, plate motion.

Apparent deformation can be attributed to a wide variety of sources, including;

- antenna modelling errors
- antenna failure
- incorrect antenna height measurement
- antenna type metadata error
- change of antenna cable
- snow or debris on antenna
- receiver type or firmware metadata error
- tropospheric modelling errors (including local temporal weather effects)

- ionospheric modelling errors
- orbit modelling errors (e.g. unmodelled solar radiation pressure effects)
- seasonal multipath effects (e.g. deciduous trees near antenna)
- variable multipath (e.g. tree growth, construction near antenna)
- unmodelled geocentre motion within the reference frame

3.2 Predictable Periodic Deformation

Semi-diurnal and diurnal solid Earth and ocean tide deformations can be modelled precisely a priori, and are routinely applied for global and continental scale geodetic analyses. Higher resolution loading models are used in areas with complex coastline shapes adjacent to shallow continental shelves and with large tidal variations. Tidal effects that can be modelled, and their magnitudes are listed as follows:

Solid Earth Tide	< 550 mm	(IERS, 2010)
Ocean Tide Loading	< 180 mm	(FES2004, EOT08a)
Atmospheric Tides	< 2 mm	(RP03)

3.3 Periodic deformations that can be modelled a posteriori

Many periodic deformations cannot be precisely modelled a priori due to high levels of temporal and spatial variability of the cause. Lower precision predictions can be used in the absence of observations using forecast models. Meterological, gravimetric, tidal and satellite altimetry observations can be used to estimate the loading effects on a specific location, and should be modelled a posteriori using real-time gravimetric data and meteorological observations for the highest precision geodetic analysis.

Atmospheric loading (non-tidal components)	< 10 mm
Hydrological loading (e.g. monsoon, dense snow cover)	< 30 mm
Non-tidal ocean loading (e.g. storm surges)	< 10 mm

3.4 Site specific deformation

Many periodic or episodic deformation effects are attributable to site-specific deformation, particularly for geodetic antennae and monuments not fixed directly to bedrock. The magnitude of these deformation effects can be estimated by precision site tie surveys, redundant CORS arrays and continued integrity monitoring. If site deformation is not quantified it becomes an error in the context of wider-field deformation modelling.

- Thermal expansion of monument (e.g. antenna mast or building)
- Wind shear of monument or building
- Surface creep (for monuments on steep slopes with a thick sub-soil horizon)
- Landslip and lateral spreading resulting from liquefaction
- Soil compaction (bedding down of large structures)
- Subsidence (water extraction, underground mining, sink hole collapse)

- Variability in sub-soil moisture (expansion and contraction)
- Permafrost decay
- Monument or antenna mount instability (gradual collapse)
- Physical disturbance of the monument or antenna

3.5 Secular geophysical deformation

Secular geophysical deformation is considered to be linear and predictable over long periods of time (> 100 yr). The dominant secular deformation mode is that associated with rigid plate motion. Velocities for sites located within the interior of rigid tectonic plates are highly linear for periods of up to thousands of years. Within plate boundary zones however, a cycle of shear strain accumulation and release (seismic activity) arising from the relative movement and locking of adjoining plates results in non-linearity of a site's time-series. High geodetic strain rates can rapidly distort geodetic networks. In regions that have been subject to ice sheet loading effects during the last glacial maximum, GIA can be significant, however this effect is also considered to be stable over periods of hundreds of years. Secular deformation tends to be widely distributed spatially and can be modelled with high precision within a local reference frame. Different modes of secular deformation, their magnitude, stability and extent are listed in Table 1.

Cause	Deformation ($mmyr^{-1}$)	Stability (yrs)	Extent (km)
Rigid plate motion	< 100	10,000	300-8,000
Tectonic Uplift	< 10	1,000	50-1,000
Glacial Isostatic Adjustment	< 10	1,000	500-3,000
Interseismic strain	< 50	100	50-500
Diapirism	< 20	1,000	20-200

Table 1. Modes of secular deformation, their magnitude, duration and extent

3.6 Non-secular geophysical deformation

Non-secular deformation is by definition episodic and non-linear in nature (e.g., seismic and volcanic deformation) and is also highly localised, particularly near fault scarps and locations of volcanic activity. Aftershock deformation, postseismic decay and slow-slip events also add further complexity to the non-secular deformation field. The high spatial variability of these effects can warrant permanent changes to, or adjustments of, a local reference frame, if the deformation exceeds positioning and dimensional tolerances in affected areas. Coordinate updates also ensure that conformity is maintained in areas of significant deformation (e.g. fault scarps across property boundaries and service easements). Large earthquakes (Mw > 8.0) can also result in observable far-field deformation up to 5,000 km from the earthquake epicentre. The magnitude of viscoelastic relaxation from these large earthquakes can exceed that of the coseismic deformation and can take place over periods of up to several decades. Whether this far-field deformation and postseismic relaxation should result in permanent coordinate changes depends upon the magnitude and tolerance of the local reference frame affected by it, and is discussed in more detail later. Table 2 shows different non-secular deformation modes.

Cause	Deformation ($mmyr^{-1}$)	Stability (yrs)	Extent (km)
Water table changes	< 250	10	<200
Volcanism	< 10,000	0.01	<30
Coseismic Deformation	< 10,000	0.00001	<1000
Slow-slip Deformation	< 300	0.1	<100
Postseismic Deformation	< 1,000	10	<1000

Table 2. Non-secular deformation effects, their magnitude, duration and extent

Other deformation events tend to be more localised, e.g. landslips, subsidence resulting from mining and ground water changes which can be particularly significant in sedimentary basins and urban areas.

4. DEFORMATION TOLERANCE LIMITS FOR POSITIONING

To what degree of precision deformation should be modelled, depends upon the tolerance requirements of the end user. For example, personal navigation typically requires a precision of a few metres at best, routinely attainable with an inexpensive handheld GNSS receiver in Single Point Positioining (SPP) mode. At this level of precision, no deformation modelling or transformation is usually necessary provided that the underlying map base is referenced to an ITRF aligned local reference frame, realised within the previous fifty years or so. On the other hand, high precision deformation monitoring using GNSS techniques may have a tolerance of just a few mm (e.g. structural engineering). Ideally, deformation monitoring surveys should be initially constraint free and so kinematic ITRF should be used as a minimally constrained datum for initial deformation analysis. Other surveys are generally concerned with positioning rather than deformation monitoring and a different approach is required.

4.1 The distinction between dimensional and positional tolerance

Dimensional tolerances are usually governed by engineering requirements (e.g. bridge and dam construction, structural engineering, plant facilities) and cadastral boundaries. Dimensions are essentially datum free, although cadastral surveys often refer to a datum as a monument or pair of monuments which define the orientation and location of a nearby boundary. Positional tolerances on the other hand are datum dependent, with any dimensions derived from two positions. Dimensional uncertainty in this instance is a function of the positional uncertainty of each location. In many cadastral and engineering surveys, dimensional tolerances override positioning tolerances with regard to external connections to a geodetic datum.

With the advent of precision GNSS positioning there is a grey area between dimensional or Local Uncertainty (LU) and Positional Uncertainty (PU). These concepts are discussed more completely in Roberts, (2009). In practical terms, LU can be described as the dimensional uncertainty between any adjoining points (e.g. two points on a bridge span, or two adjacent corners of a cadastral boundary), whereas PU is the uncertainty of a group of related points (e.g. a cadastral parcel, or structure) with respect to another group of points (not directly connected by a cadastral boundary or engineering structure) or a geodetic datum.

Linear dimensional tolerances are often described in different ways. A linear ratio is commonly used (e.g. 1 in 10,000), however parts-per-million (ppm) is also widespread. Geodetic strain rates are typically defined as a rate of change per unit length per year and are usually described in terms of $1 \cdot E^{-9} yr^{-1}$. The relationship between the different terms is as follows:

linear ratio = 1,000,000/ppm   linear ratio yr^{-1} = 1,000,000,000 / strain rate
ppm = 1,000,000/linear ratio   ppm yr^{-1} = strain rate / 1,000
strain rate = ppm $yr^{-1} \cdot$1,000 or 1,000,000,000 / linear ratio yr^{-1}

Tables 3 and 4 show typical dimensional and positional tolerances for a variety of spatial applications.

Application	Local Uncertainty (LU)			typical maximum project dimension (m)	Equivalent Positional Uncertainty at project extents (mm)
	fixed precision component (mm)	Linear Component			
		in ppm	as a ratio		
Engineering and Construction					
High precision engineering	2	10	100,000	100	5
Structural engineering	3	20	50,000	500	15
Civil Engineering	10	200	5,000	1,000	200
Civil Earthworks	100	400	2,500	1,000	500
Cadastral Surveying and Easements					
CBD Cadastral (Multi-story)	5	15	75,000	100	10
Urban Cadastral (Suburban)	15	40	25,000	200	25
Rural Residential Cadastral	25	50	20,000	1,000	75
Rural Cadastral	50	50	20,000	2,000	150

Table 3. Indicative horizontal dimensional tolerances and uncertainties (fixed and ppm component) at 1σ

Application	Positional Uncertainty (PU) (mm)
Precision Agriculture (inter-row steering)	25
Automated Mining	50
Feature survey and Site Plan 1:250 scale	50
Automated Driving	100
Underground utility maps 1:500 scale	100
Airborne Laser scanning (LiDar)	100
DCDB / Urban Services maps 1:1,000 – Asset Mapping	200
Outer-urban services GIS/maps 1:2,500 scale	500
Aircraft Instrument Landing	1,000
Suburban planning GIS/maps 1:5,000 scale	1,000
City GIS/Maps 1:10,000 scale	2,000
Personal Navigation (e.g. car)	5,000
GIS/Topographic Maps 1:25,000 scale	5,000
GIS/Topographic Maps 1:50,000 scale	10,000

Table 4. Indicative Positional Uncertainty (PU) tolerances at 1σ

4.2 Inter-relationship between tolerance requirements, deformation rates and positioning technique

For surveys where dimensional tolerances are more critical than positional tolerances (with respect to a national geodetic datum), terrestrial surveying techniques are more likely to be used (e.g. total station, terrestrial laser scanning, precise levels etc.). The use of GNSS techniques such as RTK for these surveys is usually limited to lower precision positioning (e.g. DTM spot levels, earthworks and excavation set-out) and is also likely to be localised (by local site transformation) to ensure consistency between spatial reference systems used for engineering design, terrestrial surveying and GNSS techniques.

Within tectonically stable regions, geodetic strain rates are typically less than $1E^{-9}yr^{-1}$, which implies that the life span of a local reference frame is typically hundreds of years in terms of the most stringent dimensional tolerance specifications. The continental part of the Australian Plate for example, is highly stable with strain rates of $< 0.1\ E^{-9}yr^{-1}$ which equates to a dimensional stability of 1:10,000,000,000. Intraplate earthquakes are relatively uncommon and any associated deformation with these is usually highly localised. The largest contributor to intraplate deformation is likely to be far-field deformation associated with large earthquakes within the plate boundary zone.

The most significant deformation effect of local reference frames fixed to stable tectonic plates is that of rigid plate rotation, which can be quite significant (Fig. 2). The magnitude of the effect of rigid plate rotation on fixed epoch GNSS processing is shown in Table 5.

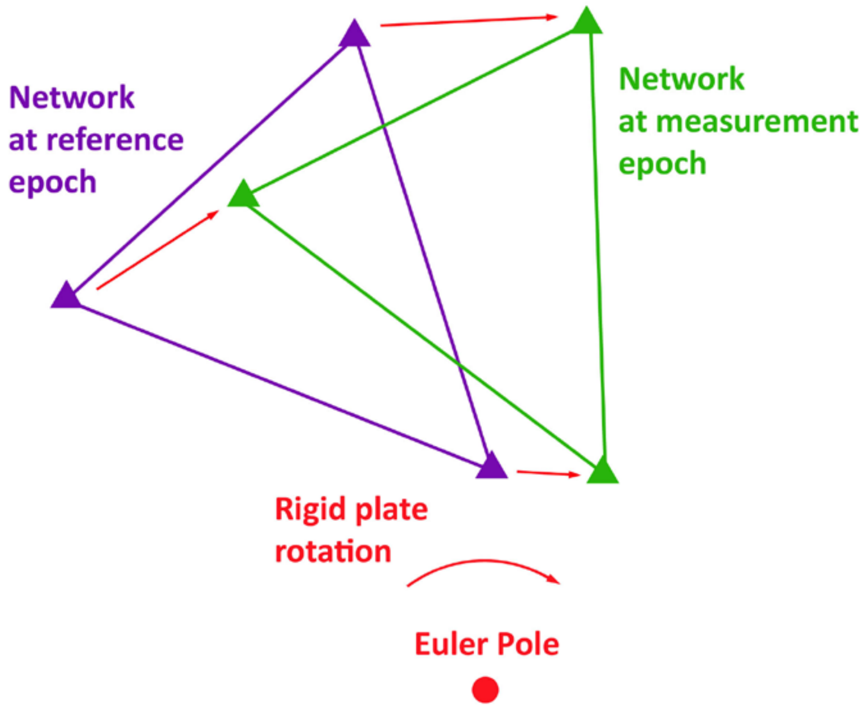

Figure 2. Schematic rigid plate rotation of network about Euler pole

Tectonic Plate	Rotation Rate °/Ma	Number of years before 15 mm PU for rover GNSS exceeded at 30 km range from CORS
Pacific	0.68	42
Australian	0.63	45
Eurasian	0.26	110
North American	0.19	151
South American	0.12	239

Table 5. Number of years till PU tolerance exceeded as a result of rigid plate rotation on a GNSS baseline

Many smaller microplates and rigid crustal blocks in deforming zones have even faster rates of rotation (e.g. 8°/My for the South Bismarck Plate in Papua New Guinea), so spans of less than 3 years can introduce 15 mm errors in baseline processing if the effect of rigid plate motion is ignored.

Assuming that a rotating tectonic plate is internally rigid, site velocities can be computed precisely using the Euler pole definition for the rigid plate (discussed in more detail later) to mitigate the effect of unmodelled rotation on baseline processing. Alternatively, a 12 or 14 parameter transformation can be used to transform between kinematic ITRF and epoch fixed ITRF to enable longer GNSS baselines to be processed correctly.

Within deforming zones, secular geodetic strain rates rarely exceed $500E^{-9}yr^{-1}$. Figure 3 plots the number of years before different survey tolerances are exceeded within a deforming zone.

Figure 4 shows the relationship between PU and deformation rates for different specifications of PU at a range of 30 km (a typical maximum baseline length between a rover GNSS and CORS station).

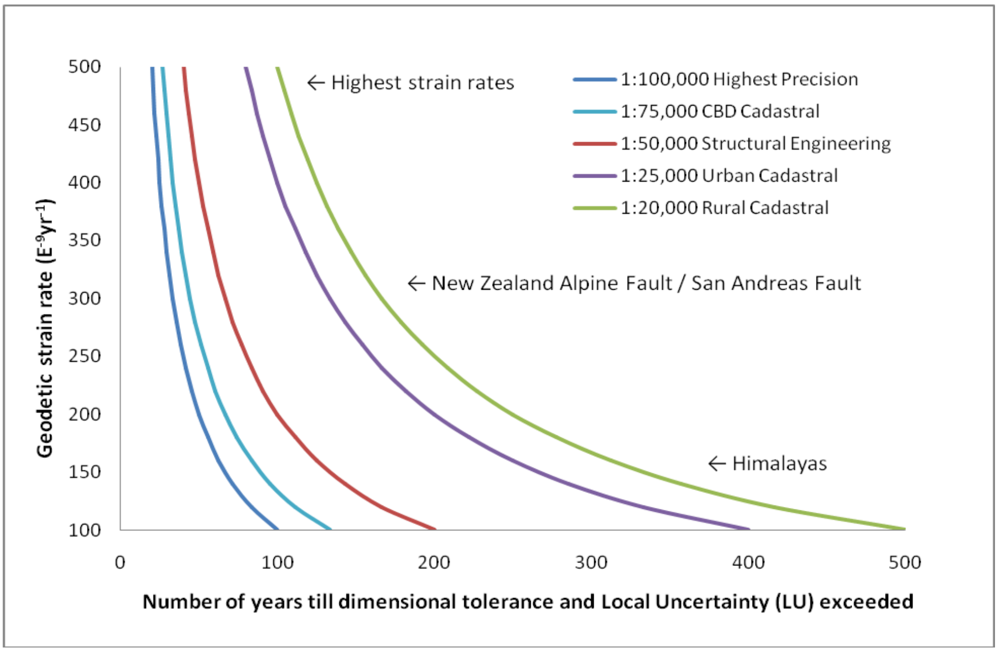

Figure 3. Number of years till LU tolerances are exceeded for differing strain rates and specifications

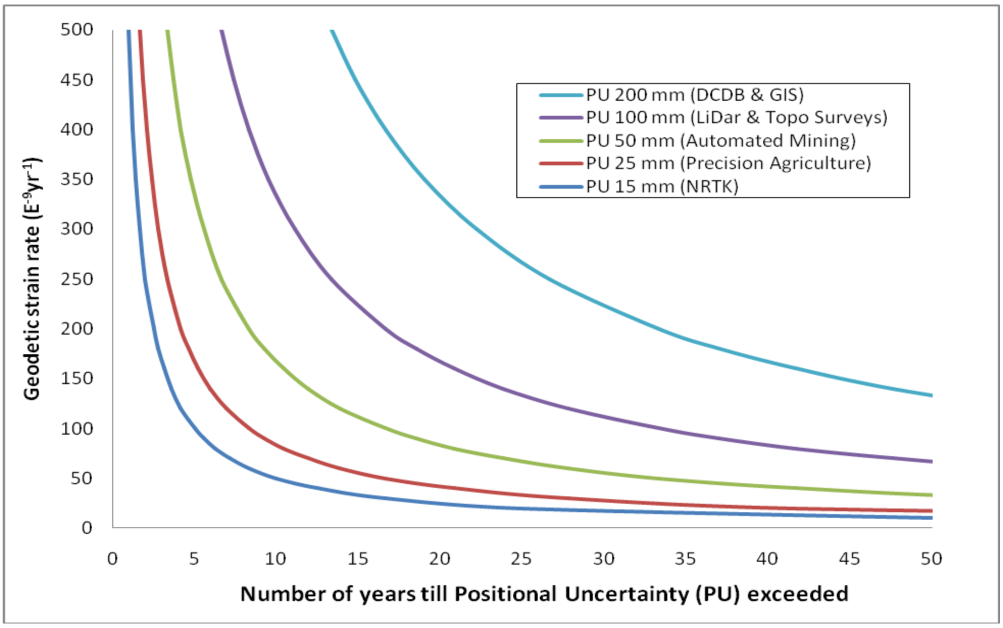

Figure 4. Number of years till PU tolerances are exceeded for differing strain rates and specifications

For example, in New Zealand, geodetic strain rates within the plate boundary zone (e.g. along the Alpine fault) are typically $300E^{-9}yr^{-1}$, which means that for local control surveys a redefinition of the local datum is required every 70 years during the interseismic period if a LU of 1:50,000 is to be maintained. Cadastral surveys would need to be redefined dimensionally every 170 years or so in order to maintain 1:20,000 precision in the absence of any seismic deformation. In order to maintain the desired 15 mm PU tolerance on a typical 30 km baseline (0.5 ppm) a deformation model is required for any epoch difference greater than a year to support CORS NRTK operations.

Realistic static GNSS tolerances are 5 mm + 0.5 ppm and NRTK uncertainties are usually 15 mm within an NRTK cell with 60 km spacing between CORS. From this we can deduce how long an NRTK cell can use fixed coordinates for its constituent CORS stations before a deformation model is required to be used (Table 6).

geodetic strain rate ($E^{-9}yr^{-1}$)	Typical deformation setting (fast-moving strike-slip plate boundary) (Note: as an illustrative guide only, as strain rate is highly variable)	Number of years before 15 mm PU for rover GNSS exceeded at 30 km range from CORS
0.5	interior of rigid tectonic plate	1000.0
5	diffuse deformation zones	100.0
10	between 300-400 km from plate boundary	50.0
20	between 200-300 km from plate boundary	25.0
50	between 150-200 km from plate boundary	10.0
100	between 100-150 km from plate boundary	5.0
200	between 50-100 km from plate boundary	2.5
500	within 50 km of fast-moving plate boundary	1.0

Table 6. Number of years till NRTK positioning tolerance is exceeded as a result of geodetic strain

Sudden or episodic localised deformation should result in the readjustment of coordinates of geodetic infrastructure in order to prevent dimensional tolerances from being exceeded with subsequent use of unadjusted coordinates. Furthermore, high resolution GIS, DCDB and spatial models of cadastral boundaries, urban services and infrastructure should reflect reality especially in instances where fault ruptures occur. For example, two cadastral reference marks or geodetic monuments reference a cadastral parcel. A major earthquake results in a large displacement of the boundary between two corners (Figure 5). To distinguish seismically affected coordinates from pre-earthquake coordinates an adjustment of the local geodetic network is required and can be implemented in the form of a gridded non-secular deformation patch. This process is described in more detail in the next section.

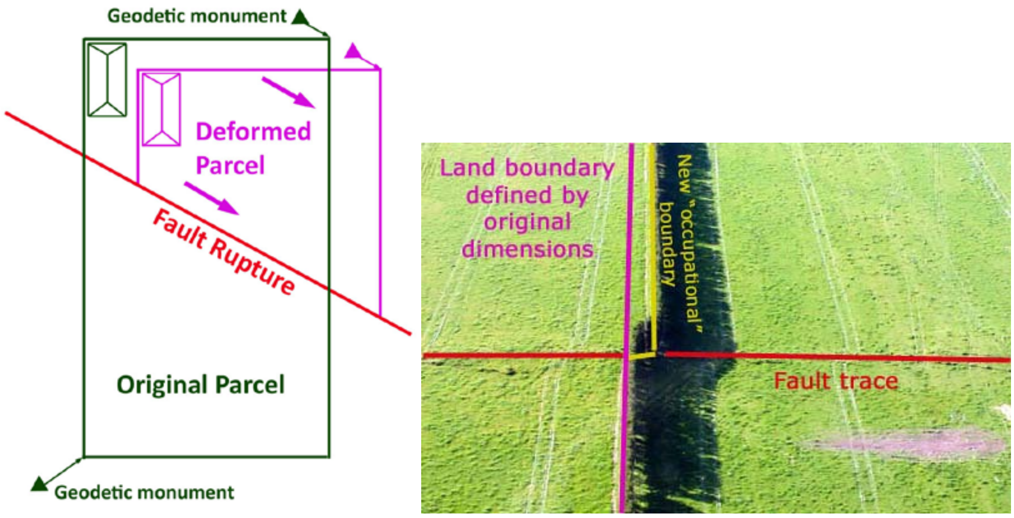

Figure 5. Effect of coseismic deformation on the cadastre and geodetic network.

5. DEFORMATION MODEL FORMAT

Deformation models can comprise two components; 1. Secular interseismic deformation, and 2. Non-secular deformation. As described previously, the secular model is used to compute interseismic deformation within ITRF and supports a static local reference frame. The non-secular deformation (patch) model represents an update or adjustment of a local reference frame to account for localised deformation, and is usually applied after a discrete event such as an earthquake results in highly localised deformation.

5.1 Format of a secular deformation model

Within rigid plate zones, a 14 parameter transformation model can be used to transform between ITRF and a local reference frame, or alternatively, the Cartesian equations that define rigid plate rotation can be applied. Site velocities for locations on a rigid rotating plate can be estimated precisely using equation (1) (Stanaway and Roberts, 2009) where ($\dot{X}, \dot{Y}, \dot{Z}$ in metres) is the ITRF site velocity in Cartesian format, (*X, Y, Z* in metres) is a location on a rigid plate defined by a rigid plate rotation model (Ω_X, Ω_Y, Ω_Z in radians per million year):

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} \Omega_Y Z - \Omega_Z Y \\ \Omega_Z X - \Omega_X Z \\ \Omega_X Y - \Omega_Y X \end{bmatrix} \cdot 1\text{E-6} \qquad (1)$$

This strategy cannot be used accurately within non-rigid components of the plate, especially if there is large difference between the measurement and reference epochs.

Site velocities in plate boundary zones can be estimated by interpretation of fault locking models such as DEFNODE (McCaffrey, 2011). Provided that the model is well constrained by inversion of observed site velocities and other geophysical observations such as earthquake slip-vectors, predicted velocities are usually precise enough for most spatial analysis applications within the interseismic period. The model can be updated regularly to account for improvements in the site velocity field arising from longer time-series of CORS and repeat observations of dense networks of passive geodetic monuments within the deforming zone.

A gridded data model similar in format to global geoid models can be used to define interseismic site velocities, and may be better suited to existing geodetic software algorithms. This model structure is currently used with the the New Zealand Deformation Model (NZDM) (Beavan and Haines, 2001; Blick *et al*., 2005). A gridded data format consists of four components:

Latitude (decimal degrees), Longitude (decimal degrees), Velocity (North), Velocity (East).

For each grid point, the site velocity components are computed by;

a. rigid-plate model
b. rigid-plate and fault-locking model
c. krigging of a dense field of observed site velocities

Site velocities for any given location are then computed by bilinear interpolation of the model. In polar regions (e.g. above 80° latitude) a stereographic projection model can be used if required to overcome the limitation of merdian convergence of a gridded model using ellipsoidal coordinates.

To ascertain what effect a planar assumption of the deformation model within a spherical system is, site velocities were computed for different model resolutions using both the Cartesian plate model directly and bilinear interpolation of site velocities for data grid points computed by the Cartesian plate model. For a 1 degree grid, the interpolated and directly computed velocities agree with a precision of better than 0.01 mm. For a 10 degree grid the agreement degrades to precision of 0.12 mm compared with a rigid plate model.

A model of global coverage with a grid spacing of 1 degree (~112 km spacing) would be 1.7 MB in size. A 0.1 degree model (~11 km spacing) would be approximately 187 MB in size and a 0.01 degree model (~1 km spacing) would be 1GB in size. To facilitate both localised use and application of the model in zones of more complex deformation requiring a higher resolution model, the following approaches can be used: For smaller regions, a grid model can be extracted from the high resolution model. This procedure is already commonplace with high resolution global geoid models such as EGM2008 for example. A 0.1 degree model covering New Zealand would be less than 0.5 MB in size.

In more complex deformation zones, where geodetic strain rates are higher, a nested model approach can be used where very high resolution (e.g. 0.01° or 0.001°) localised models can be used together with a lower resolution model. The higher resolution model can either provide a supplementary term to the site velocity interpolated from the lower resolution model, or alternatively the higher resolution model can embed the full magnitude of the velocity field. A plot of geodetic strain rates can indicate what resolution deformation model is required to support different user requirements and tolerance limits.

5.2 Format of a non-secular deformation model (patch)

The source data for modelling of an earthquake patch can be derived from a variety of different sources, typically; slip dislocation modelling, InSAR, analysis of high resolution imagery or LiDar, campaign GNSS/GPS re-observations over a dense geodetic network and terrestrial measurements.

A model of non-secular deformation arising from earthquakes and other localised deformation can be in the same format and use the same interpolation strategy as the secular deformation model. A time-tagged nested model structure is required in order to capture the full resolution of any sesimic deformation. Model metadata should also define which epochs the localised deformation is defined between (e.g. 2000.0 and 2011.153).

For example, a 1 degree model would be used for far-field deformation arising from Mw8.0 - 9.5 earthquakes and associated post-seismic deformation (e.g. viscoelastic relaxation), whereas a 0.00001 degree model would be used to map actual fault scarps at 1 metre resolution. Clearly, it is impractical to develop a global model at such a high resolution, and so a nested data structure can be used (Figure 6). Data structures for smaller locations can also be in terms of local grid coordinates (e.g. UTM) for ease of use. For a typical larger urban area of 4000 km^2, differing resolution models would have the following sizes: 1 km resolution, 110 kB, 100 m resolution 10.8 MB, 10 m resolution 1 GB. Winefield *et al.* (2010) describe how such a local deformation patch has been applied in New Zealand.

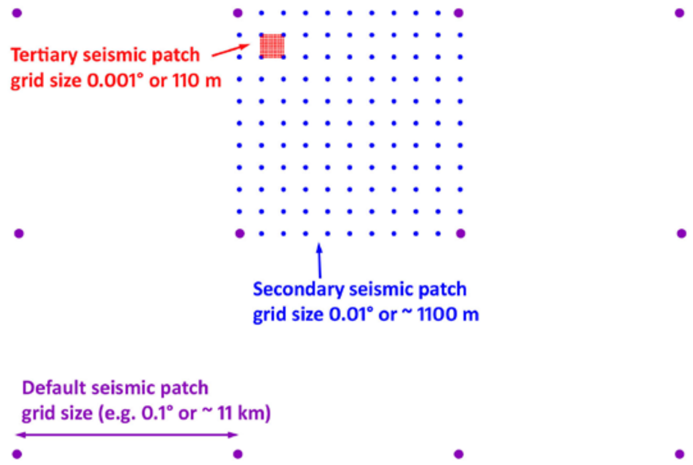

Figure 6. Nested structure of seismic patch grid model to accommodate localised deformation

5.3 Vertical Deformation Models

As is the case with non-secular deformation, vertical deformation is highly variable in the temporal and spatial domain, although generally smaller in magnitude. Stable bedrock locations within the interior of rigid tectonic plates have vertical deformation rates close to zero (or less than 0.2 $mmyr^{-1}$), however within plate boundary zones, vertical deformation rates can be as high as 10 $mmyr^{-1}$ due to tectonic processes. In regions such as Scandinavia and Canada, uplift arising from GIA can also be as much as 10 $mmyr^{-1}$. In more localised areas, vertical deformation is usually associated with changes in groundwater and mining operations.

How vertical deformation is handled in terms of modelling depends upon the secularity of the deformation. Tectonic and post glacial deformation is typically highly secular in character and a 1 degree model of vertical rates can be used to model and predict elevations to support GNSS processing and analysis for example. Such a model would be interpolated and used in the same way as a horizontal deformation model. In regions where subsidence is occurring as a result of groundwater changes, for example, the vertical rate is likely to vary over much shorter time periods, however a higher resolution model can be used and updated as often as required. Alternatively, vertical deformation can be treated in a non-secular way with regular patch updates everytime a tolerance limit is reached, or earthquake occurs.

Hydrological analysis and engineering projects sensitive to elevation (e.g. drainage), and CORS networks require high precision vertical deformation models as the classical approach of holding benchmark elevations fixed in areas subject to vertical deformation can compromise the integrity of these projects.

6. DEFORMATION MODELS IN PRACTICE

As described previously, deformation models to support spatial applications ideally should have secular (interseismic) and non-secular (episodic or patch) components. The secular model is used to propagate kinematic ITRF coordinates to the specified epoch of a local reference system so that local coordinates appear to be static even though they are subject to larger scale deformation. New Zealand has used this approach since 2000 with the adoption of NZGD2000, which incorporates a deformation model, NZDM (Blick *et al.*, 2005).

A non-secular, or seismic patch model is a sum of all episodic offsets between the reference and measurement epochs. This model is invoked in much the same way as a grid distortion model, and quantifies permanent deformation of the original geodetic network in excess of any interseismic deformation since the reference epoch. The patch can also incorporate small reference frame translations and biases arising from imprecisely estimated interseismic deformation models. After large earthquakes and subsequent aftershocks or slow-slip events, latency in the release of seismic patches is beneficial in order to account for any postseismic relaxation which is non-linear in character. The two models are used in conjunction with each other (Figures 7 and 8).

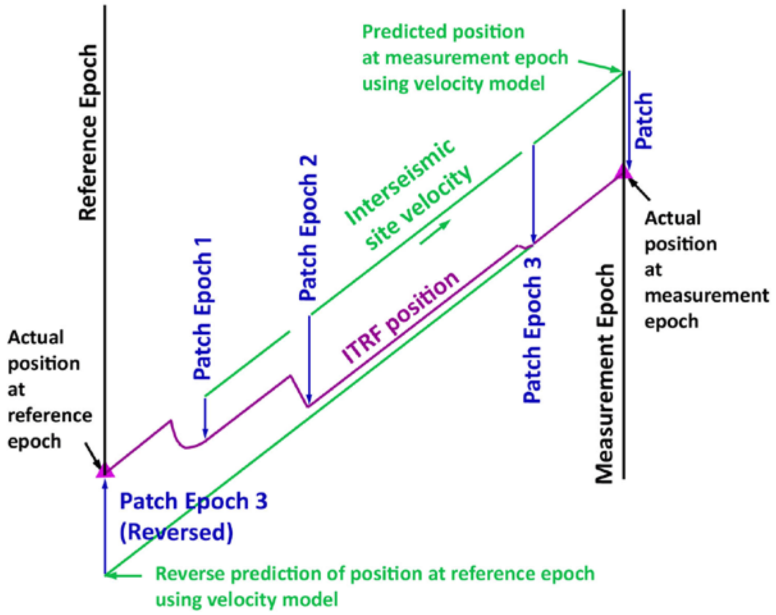

Figure 7. Schematic representation of the application of interseismic velocity models and seismic patches.

Assuming that any GNSS processing and analysis is done within a kinematic ITRF/IGS reference frame, deformation modelling propagates positions derived from this analysis into the local reference frame. Propagation equations (2 and 3) from Stanaway and Roberts, (2011) can be used to invoke the deformation models (based on flowchart in Figure 8):

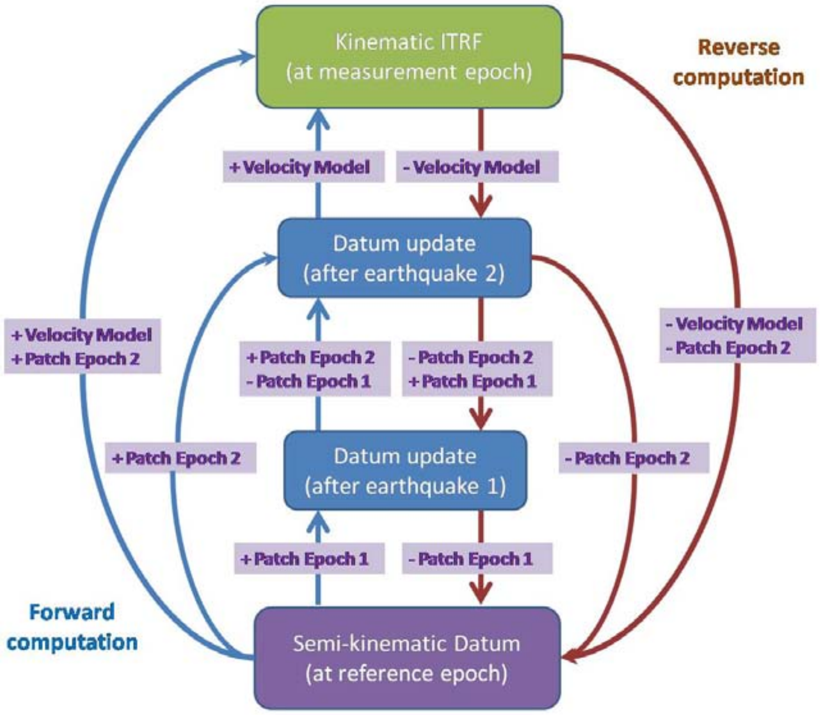

Figure 8. Flowchart showing propagation pathways between a semi-kinematic datum (fixed at the reference epoch) and kinematic ITRF.

Kinematic ITRF (typically the latest weekly realisation of the IGS reference frame) should be used as the datum for any GNSS data processing. Interpolation of the interseismic deformation model enables the reference epoch to be recovered anywhere in the network. A seismic deformation patch then accounts for any misalignment of coordinates between the interseismic model and the coordinates of the CORS at the reference epoch. Any further misalignment at the CORS can be attributed to two factors: (1) an imprecise interseismic velocity model, and (2) unmodelled deformation within the patch. These two factors can be isolated and quantified by analysis of the CORS time-series. Analysis of the time-series can identify periods of non-linear deformation (e.g. coseismic, interseismic and slow slip events). These deformations can be summed into a seismic deformation patch which should be updated after each significant seismic event. A significant event is one that results in non-secular deformation in excess of the positioning tolerances required for the datum or NRTK operation.

Semi-kinematic datum at reference epoch

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t} + \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \cdot (t_0 - t) - \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}_{PATCH} \qquad (2)$$

where,

t_0 is the reference epoch (in decimal years)
t is the epoch of measurement (in decimal years)
$(X, Y, Z)_{t_0}$ are the coordinates computed at the reference epoch (metres),
$(X, Y, Z)_t$ are the kinematic ITRF coordinates at the measurement epoch (in metres),
$(\dot{X}, \dot{Y}, \dot{Z})$ is the ITRF site velocity interpolated from the interseismic model (m/yr),
$(\Delta X, \Delta Y, \Delta Z)_{PATCH}$ is the accumulated seismic deformation between the reference and measurement epochs interpolated from the most up-to-date seismic patch model (in metres)

Semi-kinematic datum at epoch update (after earthquake)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_{PATCH}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t} + \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \cdot (t_0 - t) \qquad (3)$$

where,
$(X, Y, Z)_{t_{PATCH}}$ are the coordinates computed after the earthquake patch is applied (metres)

Model integrity monitoring

Deformation model integrity can be monitored at each CORS station within the network. This can be achieved by comparing the reference epoch coordinates computed using equation (2) with the reference epoch coordinates for the station. Whenever the difference between the two exceeds a specified tolerance (e.g. 15 mm) an alert can be raised. This would usually happen after an earthquake, or commencement of a slow-slip event. In the absence of any of these episodic events, the inter-seismic velocity model would need to be verified and updated by analysis of the CORS time-series. Repeat observations over a dense network of passive geodetic monuments can also be used to verify and improve the precision of the deformation models in current use.

CONCLUSIONS

Modern positioning techniques are rapidly becoming more precise and ubiquitous. As a consequence of this, unmodelled deformation of the Earth is now resulting in widespread errors in the application of precise positioning and GNSS baseline processing. Conflicts are also arising between global (kinematic) and local (static) reference frames and static geodetic datums as they become misaligned due to tectonic processed and other causes of deformation.

In this paper, a novel approach to the modelling of deformation has been described, that can work in practice. Application of a bimodal (interseismic model plus patch) deformation model allows users to transform positions between global, regional and local reference frames without significant loss of precision. The modelling strategy can be applied in PPP, GNSS post-processing and GIS to enable the full benefits of kinematic and static reference frames to be realised.

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